Satellite Constellation Orbit Design
to Enable a Space-Based Radio Interferometer

AAS 17-607

AAS/AIAA Astrodynamics Specialist Conference

Stevenson, WA, August 20-24, 2017

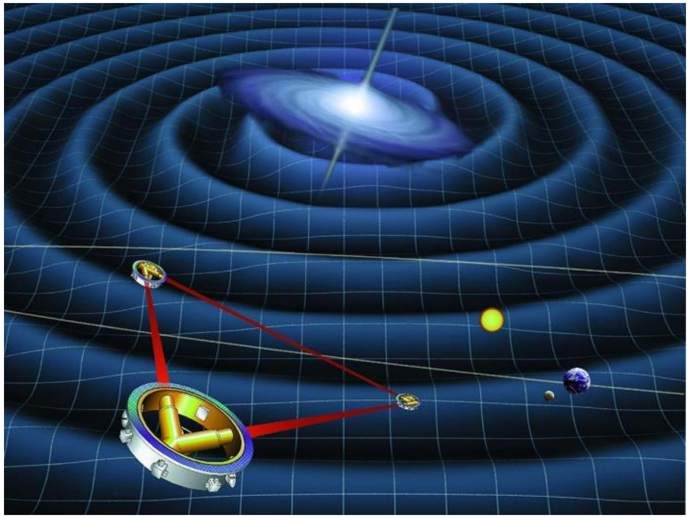
Sonia Hernandez

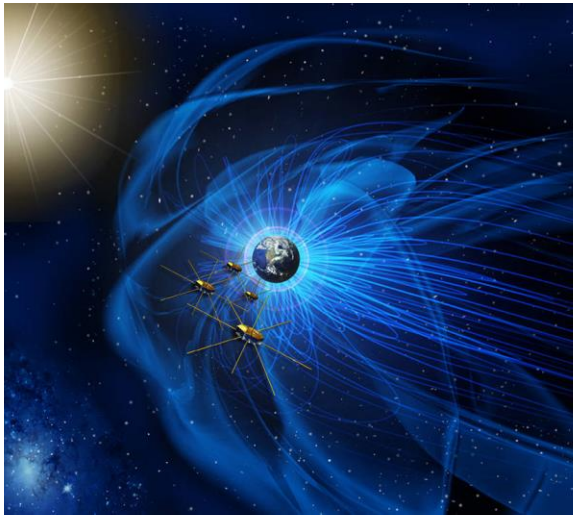
Jeffrey R. Stuart, David M. Garza, Stephen B. Broschart, Sebastian J. I. Herzig, and Steve A. Chien

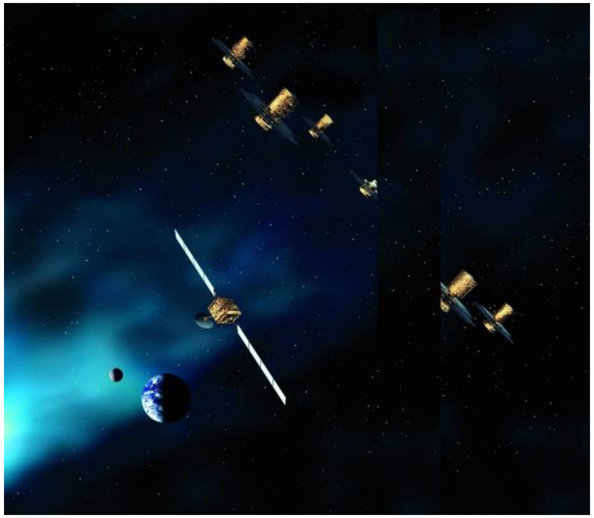
Jet Propulsion Laboratory, California Institute of Technology

© 2017 California Institute of Technology. Government sponsorship acknowledged

Introduction





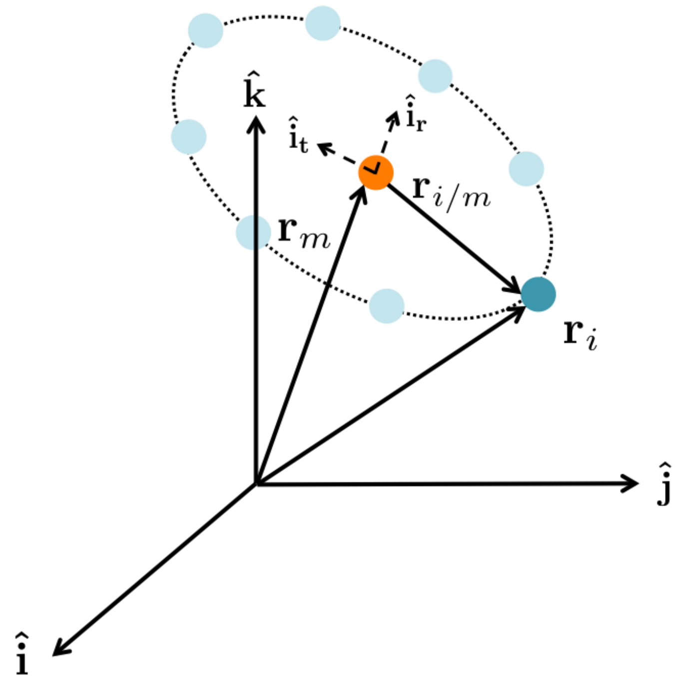


- ***Networked constellation*** missions of small spacecraft benefit from lower cost, increased robustness, and enable novel types of missions
- Advancements in ***autonomy*** make operations of constellations more manageable and less costly
- ***Space-based radio interferometers***, where a number of satellites in a (networked) constellation act as an array of radio telescopes can achieve resolutions impossible to achieve with Earth-based interferometers
- Mothership in a reference orbit can act as a ***relay*** for the constellation
- What ***technology*** exists today and what technology is still needed to enable these types of missions?

Constellation Design Methods

- Mothership (reference) spacecraft
- N daughter spacecraft
- Relative to the mothership, in a RTN frame

$$\mathbf{r}_{i/m} = \left[x_i \,\hat{\mathbf{i}}_r, y_i \,\hat{\mathbf{i}}_t, z_i \,\hat{\mathbf{i}}_n \right]^T$$
$$\mathbf{v}_{i/m} = \left[\dot{x}_i \,\hat{\mathbf{i}}_r, \dot{y}_i \,\hat{\mathbf{i}}_t, \dot{z}_i \,\hat{\mathbf{i}}_n \right]^T$$



- Any formation design strategy must begin with a ***method to predict and analyze*** both the ***absolute*** as well as ***relative*** motion of spacecraft.

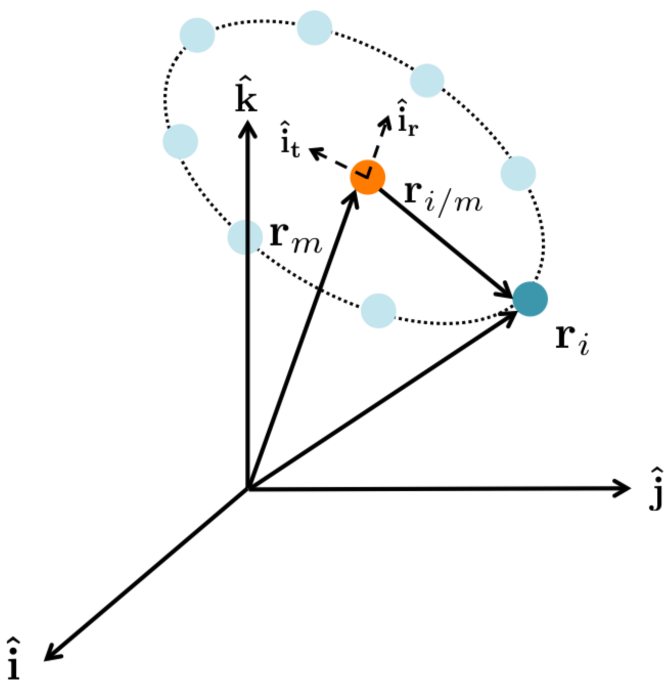
- <u>***Goal:***</u> ***Design simple and accurate design strategies that can quickly assess the best geometry design for N spacecraft constellation***

  1. **Constellation design using Linear Dynamics (CLD)**
     - Analytical framework, especially useful for initial design stages
  2. **Constellation design using Invariant Manifold (CIM)**
     - Advantageous when considering higher order dynamics

Constellation design using Linear Dynamics (CLD)

- **Clohessy-Wiltshire (CW)**
  - Assumes mothership is in a circular orbit about the central body
  - Write relative motion equations assuming two-body motion
  - Use a binomial expansion to first order, and assume $r_{i/m}$ is small

$$\boxed{\begin{aligned} \ddot{x}_i &= 2\omega\dot{y}_i + 3\omega^2 x_i \\ \ddot{y}_i &= -2\omega\dot{x}_i \\ \ddot{z}_i &= -\omega^2 z_i \end{aligned}}$$



- **Analytical Solution**

$$\begin{aligned} x_i(t) &= 2\left(2x_{i_0} + \dot{y}_{i_0}/\omega\right) - \left(3x_{i_0} + 2\dot{y}_{i_0}/\omega\right)\cos\xi + \left(\dot{x}_{i_0}/\omega\right)\sin\xi \\ y_i(t) &= \left(y_{i_0} - 2\dot{x}_{i_0}/\omega\right) - 3\boxed{\left(2x_{i_0} + \dot{y}_{i_0}/\omega\right)}\xi + \left(2\dot{x}_{i_0}/\omega\right)\cos\xi + 2(3x_{i_0} + 2\dot{y}_{i_0}/\omega)\sin\xi \quad (7) \\ z_i(t) &= z_{i_0}\cos\xi + \left(\dot{z}_{i_0}/\omega\right)\sin\xi \end{aligned}$$

$$\boxed{\xi = \omega t}$$

Constellation design using Linear Dynamics (CLD)

- **Clohessy-Wiltshire (CW)**
  - Assumes mothership is in a circular orbit about the central body
  - Write relative motion equations assuming two-body motion
  - Use a binomial expansion to first order, and assume $r_{i/m}$ is small

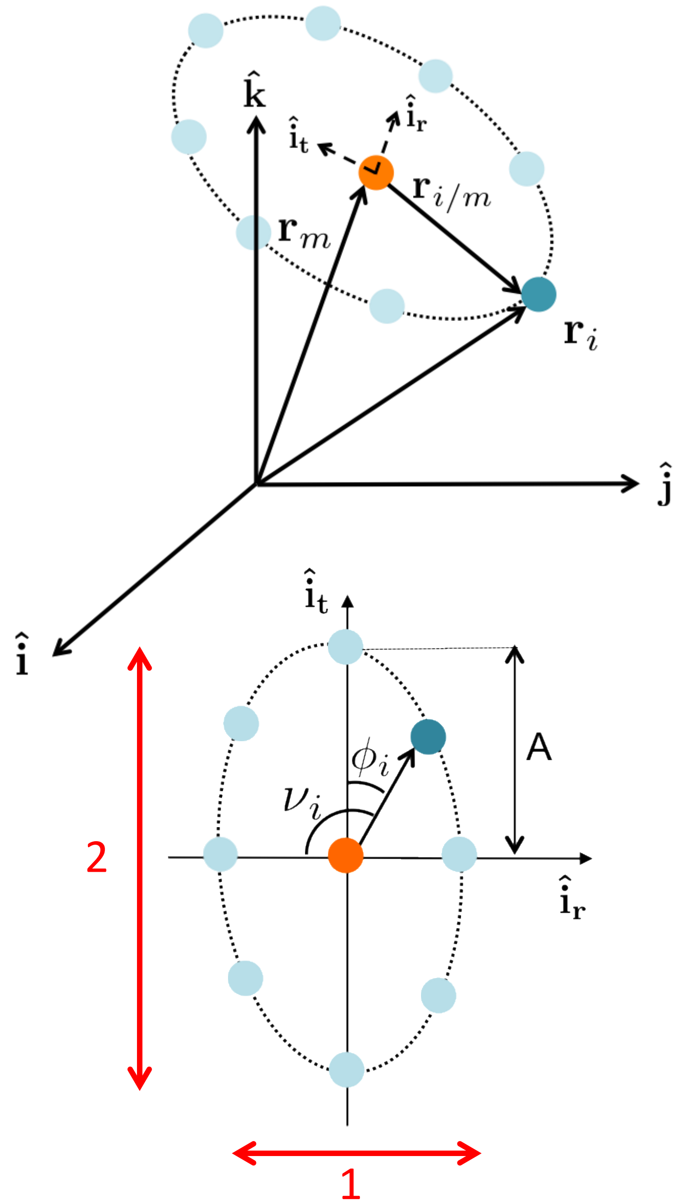
$$\boxed{\begin{aligned} \ddot{x}_i &= 2\omega\dot{y}_i + 3\omega^2 x_i \\ \ddot{y}_i &= -2\omega\dot{x}_i \\ \ddot{z}_i &= -\omega^2 z_i \end{aligned}}$$

- **Periodic Motion**

0

$$\begin{aligned} x_i(t) &= \frac{1}{2}A\sin(\alpha + \phi_i(t)) \\ y_i(t) &= A\cos(\alpha + \phi_i(t)) + y_c \\ z_i(t) &= B\sin(\beta + \phi_i(t)) \end{aligned}$$

In-plane motion is given by ***2x1*** ellipse or ***ring***, where size is a function of the ***eccentricity***



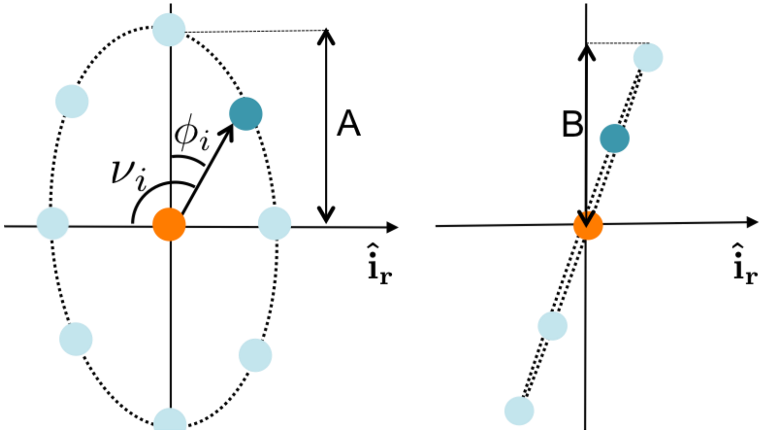
Using CLD for Constellation Design

- Relative motion for ***N spacecraft:***

  For each ring in a constellation:

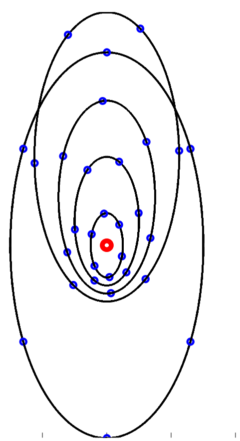
  For each spacecraft on a ring:

$$
\begin{aligned}
x_{i_j}(t) &= \frac{1}{2} A_i \sin(\phi_{i_j}(t)) \\
y_{i_j}(t) &= A_i \cos(\phi_{i_j}(t)) + y_{c_i} \\
z_{i_j}(t) &= B_i \sin(\beta_i + \phi_{i_j}(t))
\end{aligned}
$$



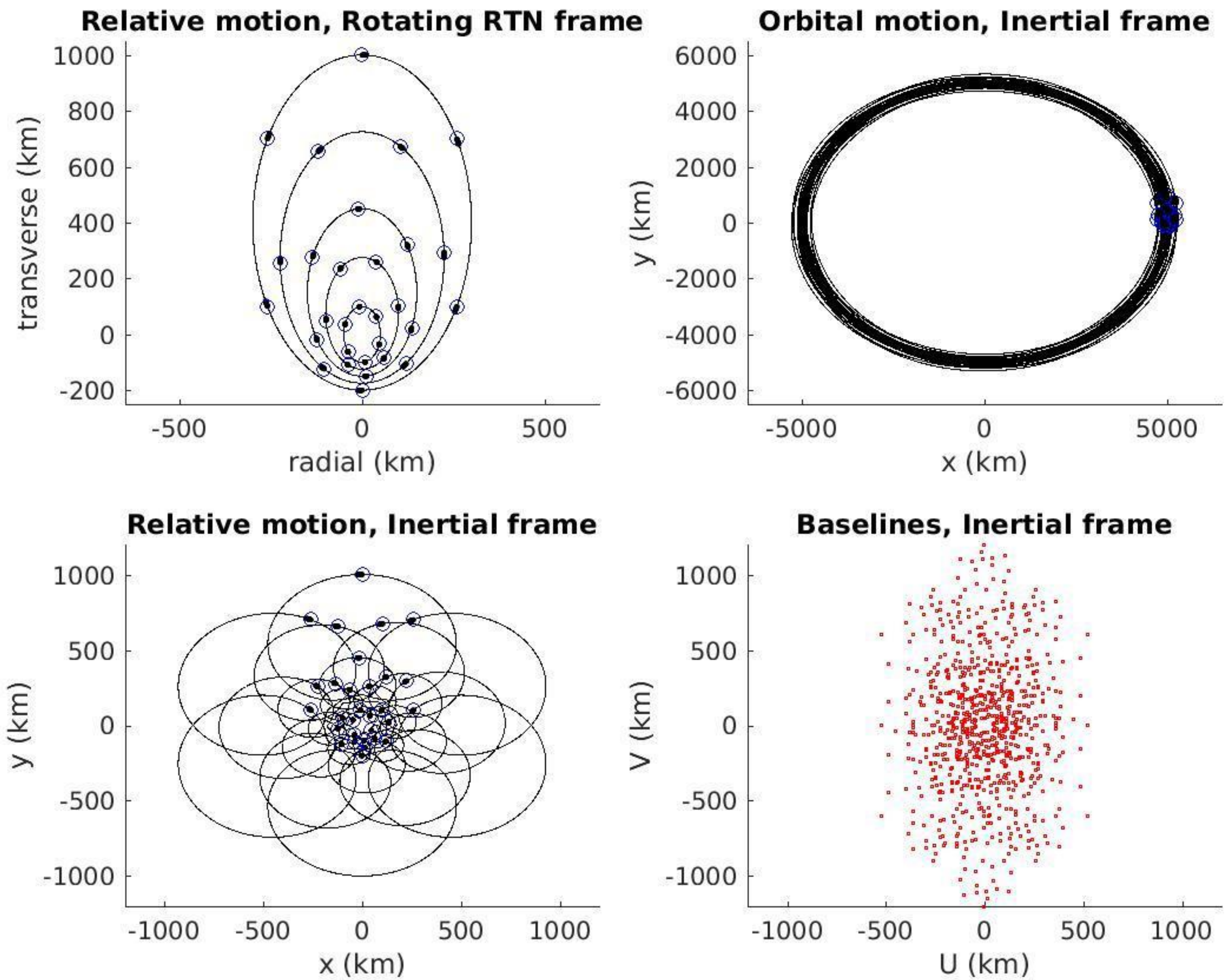
- ***5N parameters***: ***4N geometry*** parameters and ***N phasing*** parameters

Formation Parameters	Ring Parameters	
	In-plane	Out-of-plane
ω = orbit period	A = ring size	B = max. displacement
n_r = n° rings	ϕ = angular displacement	β = ring orientation
$n_{sc/r}$ = n° of spacecraft/ring	y_c = center of ring	



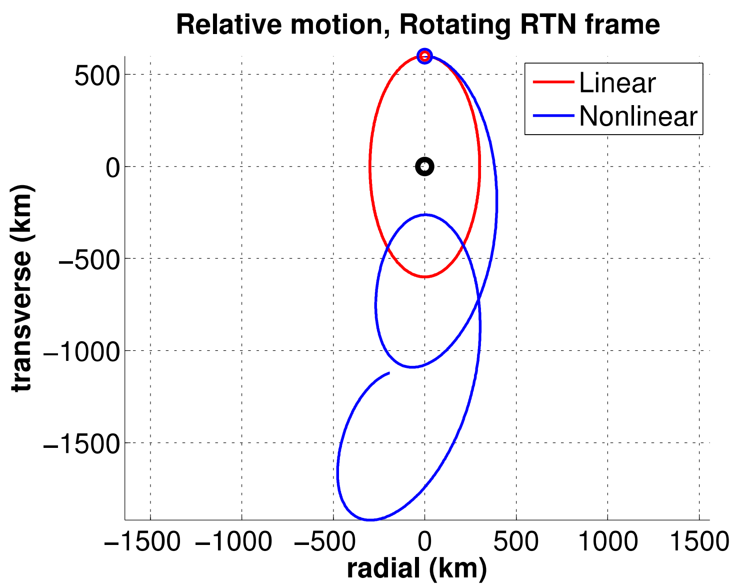
- Design ***rings*** of varying sizes and centers, mimicking a gear-like movement which allows for optimum science target (baseline) coverage

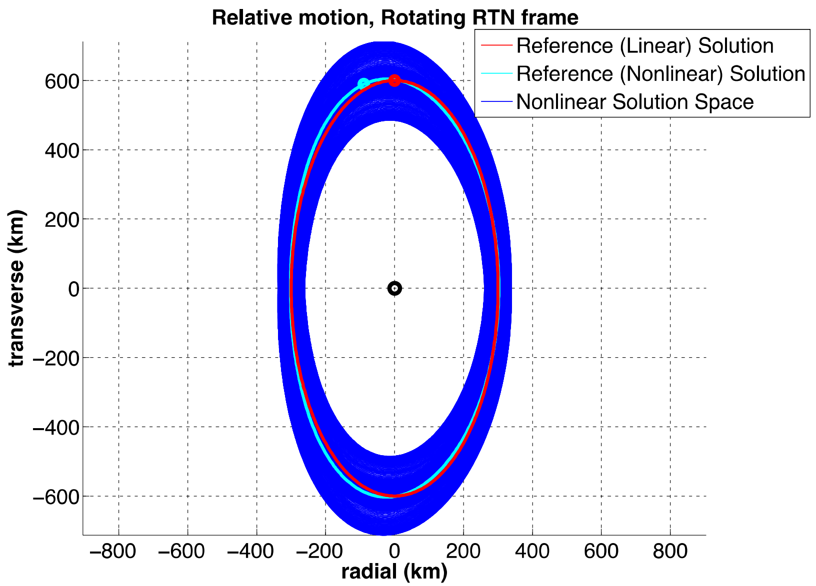
CLD: Example Constellation around Moon



Converting from Linear to Nonlinear Motion

- Fix ***semi-major axis*** to the same one as the mothership
- Depending on phase at which the conversion is performed, greater discrepancies from the linear solution might appear.

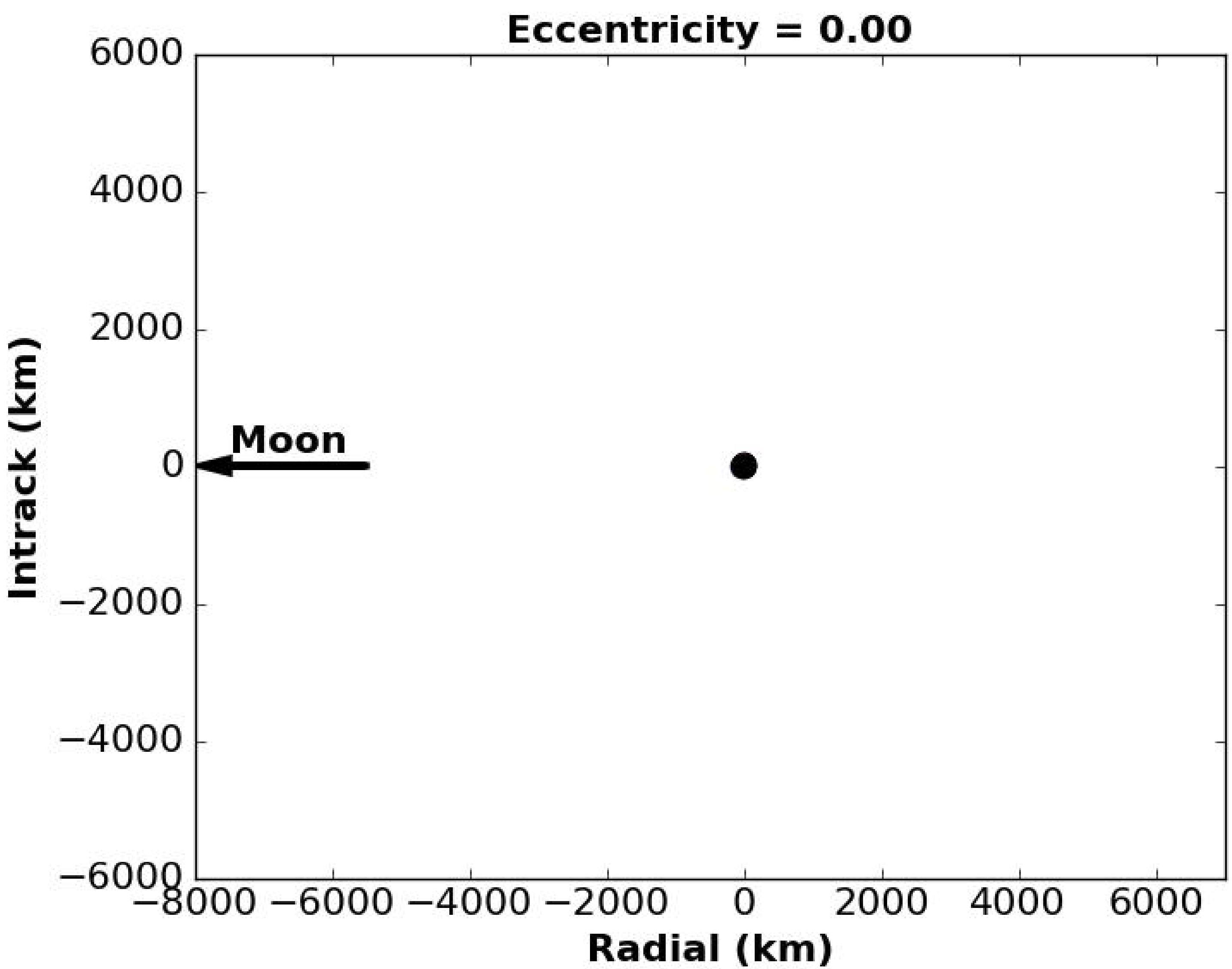




- ***Minimum deviation*** between linear and nonlinear occurs at ***φ=0°*** (ν=90°)
- Make conversion at φ=0° to obtain ring geometry, then can distribute spacecraft at any desired phase

Linear model can be used to initially design the constellation, because when converting to the ***nonlinear model*** we are able to reproduce the ***same design***

Eccentricity Variation in Nonlinear Model



2. Constellation design using Invariant Manifolds (CIM)

- Use ***dynamical system theory*** to ***excite center eigenvectors*** of the reference path in different directions to create the constellation
- Motion of daughter spacecraft can be ***linearized*** about a reference path

$$\dot{\boldsymbol{x}}_i = \boldsymbol{A}(t)\boldsymbol{x}_i \qquad \boldsymbol{A}(t) = \begin{bmatrix} \boldsymbol{0}_{3\times3} & \boldsymbol{I}_{3\times3} \\ \frac{\partial^2 U}{\partial \boldsymbol{r}_m^2} & \boldsymbol{0}_{3\times3} \end{bmatrix}$$

- Variations in initial state to final state are given by STM

$$\boldsymbol{x}_i(t) = \boldsymbol{\Phi}(t,t_0)\boldsymbol{x}_i(t_0) \qquad \dot{\boldsymbol{\Phi}}(t,t_0) = \boldsymbol{A}(t)\boldsymbol{\Phi}(t,t_0)$$

- Monodromy matrix: STM propagated for one period
  - ***Eigenvalues*** λ_i, i = 1,…,6 give stability of periodic (reference) orbit
  - ***Eigenvector*** $\mathbf{e}_i$, i = 1,…,6 are used to excite relative motion in specific directions
- Relative motion can be generated via

$$\boldsymbol{x}_i(\tau,\boldsymbol{\theta}_i,\boldsymbol{\varepsilon}_i) = \sum_{k=1}^{n} \boxed{\varepsilon_{i_k}} \Big(\cos(\theta_{i_k})\mathrm{Re}[\hat{\mathbf{e}}_k(\tau)] - \sin(\theta_{i_k})\mathrm{Im}[\hat{\mathbf{e}}_k(\tau)] \Big)$$

2. Constellation design using Invariant Manifolds (CIM)

- Use ***dynamical system theory*** to ***excite center eigenvectors*** of the reference path in different directions to create the constellation
- Motion of daughter spacecraft can be ***linearized*** about a reference path

$$\dot{\boldsymbol{x}}_i = \boldsymbol{A}(t)\boldsymbol{x}_i \qquad \boldsymbol{A}(t) = \begin{bmatrix} \mathbf{0}_{3\times3} & \boldsymbol{I}_{3\times3} \\ \frac{\partial^2 U}{\partial \boldsymbol{r}_m^2} & \mathbf{0}_{3\times3} \end{bmatrix}$$

- Variations in initial state to final state are given by STM

$$\boldsymbol{x}_i(t) = \boldsymbol{\Phi}(t,t_0)\boldsymbol{x}_i(t_0) \qquad \dot{\boldsymbol{\Phi}}(t,t_0) = \boldsymbol{A}(t)\boldsymbol{\Phi}(t,t_0)$$

- Monodromy matrix: STM propagated for one period
    - ***Eigenvalues*** λ_i, i = 1,…,6 give stability of periodic (reference) orbit
    - ***Eigenvector*** $\mathbf{e}_i$, i = 1,…,6 are used to excite relative motion in specific directions
- Relative motion can be generated via

$$\boldsymbol{x}_i(\tau,\boldsymbol{\theta}_i,\boldsymbol{\varepsilon}_i) = \sum_{k=1}^{n} \varepsilon_{i_k}\Big(\cos(\theta_{i_k})\mathrm{Re}[\hat{\mathbf{e}}_k(\tau)] - \sin(\theta_{i_k})\mathrm{Im}[\hat{\mathbf{e}}_k(\tau)]\Big)$$

- For ***two-body motion*** $|\lambda_i|=1$
    - 2 form a complex conjugate pair
    - 2 are repeated strictly real vectors (monodromy matrix degenerate)
    - 2 eigenvectors are unique and strictly real

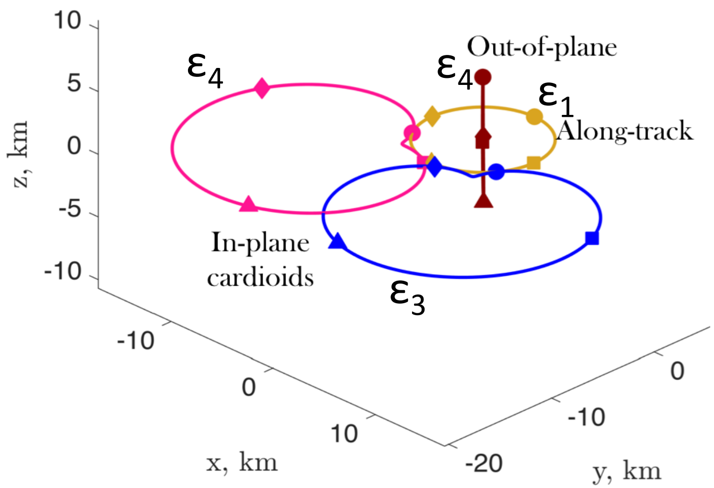
4 geometry parameters and ***1 phasing*** parameter

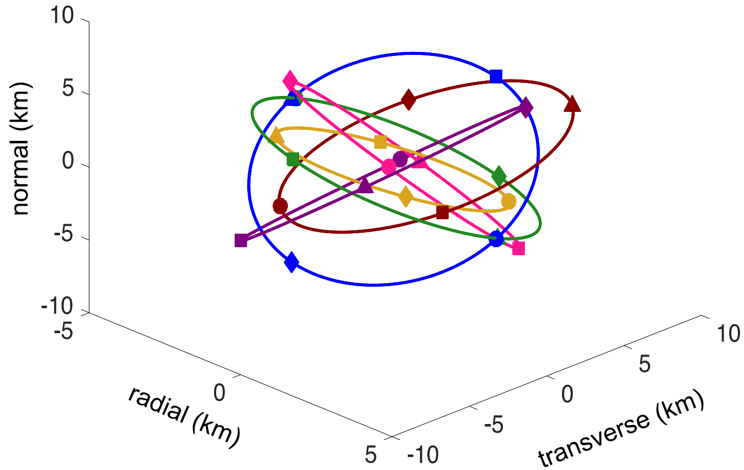
$$\boldsymbol{x}_i(\boldsymbol{\varepsilon}_i,\boldsymbol{\theta}_i) = \boxed{\varepsilon_{i_1}}\hat{\mathbf{e}}_1 + \boxed{\varepsilon_{i_2}}\hat{\mathbf{e}}_2 + \boxed{\varepsilon_{i_3}}\hat{\mathbf{e}}_3 + \boxed{\varepsilon_{i_4}}\Big(\cos(\boxed{\theta_{i_4}})\mathrm{Re}[\hat{\mathbf{e}}_4] - \sin(\theta_{i_4})\mathrm{Im}[\hat{\mathbf{e}}_4]\Big)$$

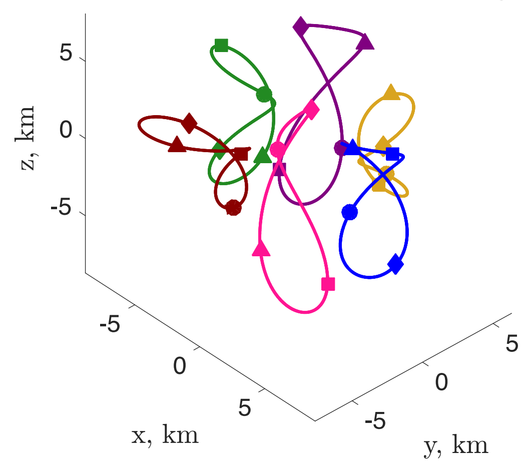
Example Constellation in GEO Graveyard using CIM

- 6 s/c constellation with the purpose of observing coronal mass ejections from the Sun
- GEO Graveyard 25 hour period

i	ϵ_{i_1} (km)	ϵ_{i_2} (km)	ϵ_{i_3} (km)	ϵ_{i_4} (km)	θ_{i_4} (deg)
1	3	0	-2	2.5	150
2	0	2	-2	2	240
3	-2.5	2	0	3	60
4	-2	0	2.5	3	180
5	0	-2	2	2	300
6	3	-2.5	0	3	120







Relative frame fixed at reference (RTN)

Relative frame inertial equatorial axis

Comparison Between CLD and CIM

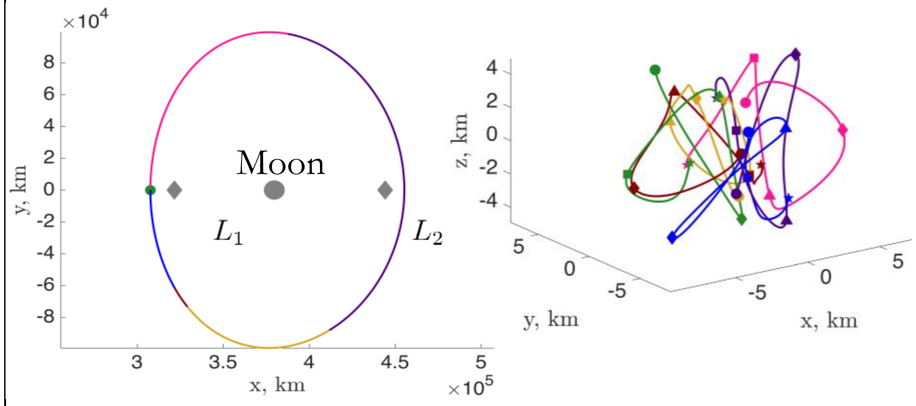
CLD

- ***Analytical*** solution exists in linear model, which is advantageous for initial design stages
- For small eccentricity, conversion to two-body model can be made while preserving the same linear geometry
- For higher order dynamics, stationkeeping costs need to be taken into account

- Constellation same period
- ***5N parameters***
  - 4N ring geometry: A, B, y_c, β
  - 1N phasing: φ

CIM

- ***Higher order dynamics*** can be used to define the constellation
- Example: ***DRO in CR3BP***

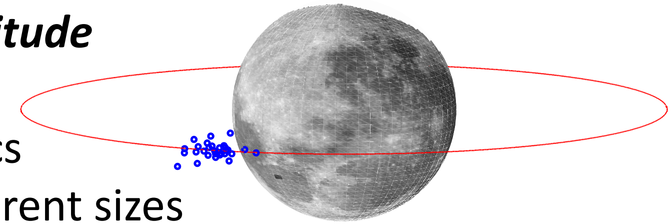


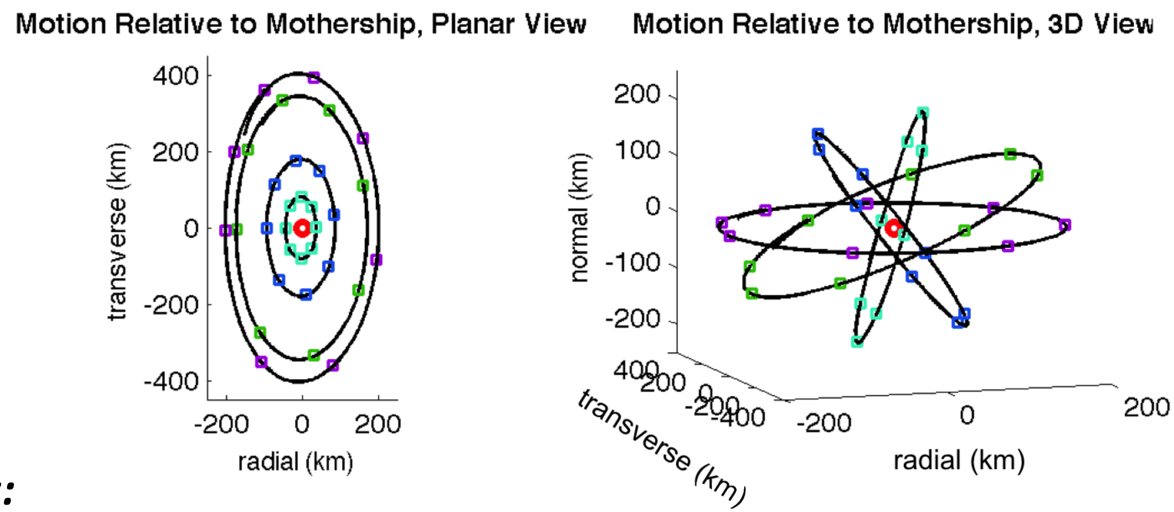
- Constellation same period
- ***5N parameters***
  - 4N ring geometry: $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$
  - 1N phasing: θ

Even though both methods are derived using ***different approaches***, the ***same constellation design*** can be achieved with either method

Lunar Orbiting Constellation Example

- Goal: Observation of ***distant galaxies*** using ***radio interferometer***
- The constellation needs to be far enough from the Earth to avoid interference
- Reference orbit at circular orbit at ***5,000 km lunar altitude***
- 600 km maximum baseline
- Use CLD for constellation design in two-body dynamics
  - ***32 daughter s/c*** distributed along ***4 rings*** of different sizes
  - 12kg small satellites, 1N thruster, Isp = 200 s





- ***Operations:***
  1. Deployment: Depart in ESPA ring from mothership, using 20 m/s
  2. Reconfiguration: 20 m/s allowed over 6 month mission time span
  3. Baseline Coverage: Observe maximum celestial sphere
  4. Stationkeeping Costs

Reconfiguration

- Reconfiguration allows for more baseline acquisition, which allows for ***more science***
- Need orbit ***period to be constant***, so constellation does not drift apart
- Maneuvering direction ***perpendicular to velocity***

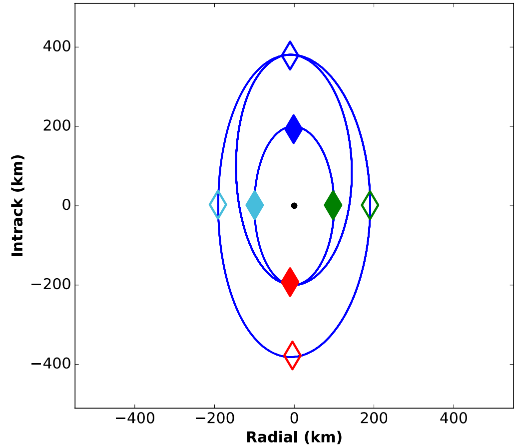
$$\ddot{\mathbf{r}}_i = -\mu \frac{\mathbf{r}_i}{r_i^3} + \mathbf{f}_i \qquad E_i = \frac{1}{2}(\mathbf{v}_i^T \mathbf{v}_i) - \mu(\mathbf{r}_i^T \mathbf{r}_i)^{-1/2} \qquad \dot{E}_i = \mathbf{v}_i^T \mathbf{f}_i = 0 \iff \mathbf{f}_i \perp \mathbf{v}_i$$

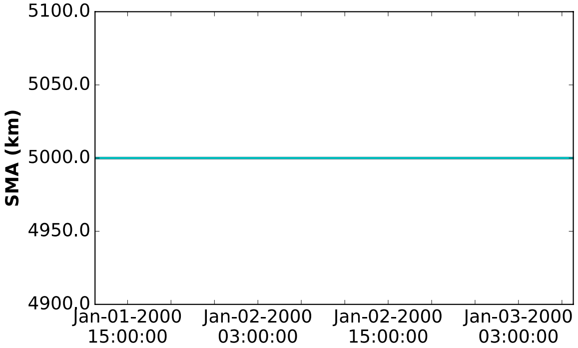
- In-plane and out-of-plane change

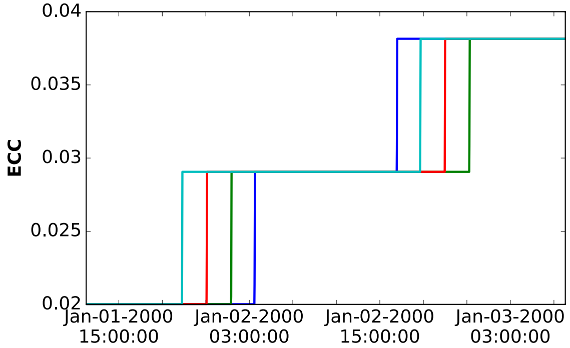
$$\boxed{\mathbf{f}_i = \frac{T}{m}\left[\frac{k_1 \hat{\mathbf{v}}_i \times \hat{\mathbf{h}}_i + k_2 \hat{\mathbf{h}}_i}{\|k_1 \hat{\mathbf{v}}_i \times \hat{\mathbf{h}}_i + k_2 \hat{\mathbf{h}}_i\|}\right]}$$

k1 in-plane thrust
k2 out-of-plane thrust

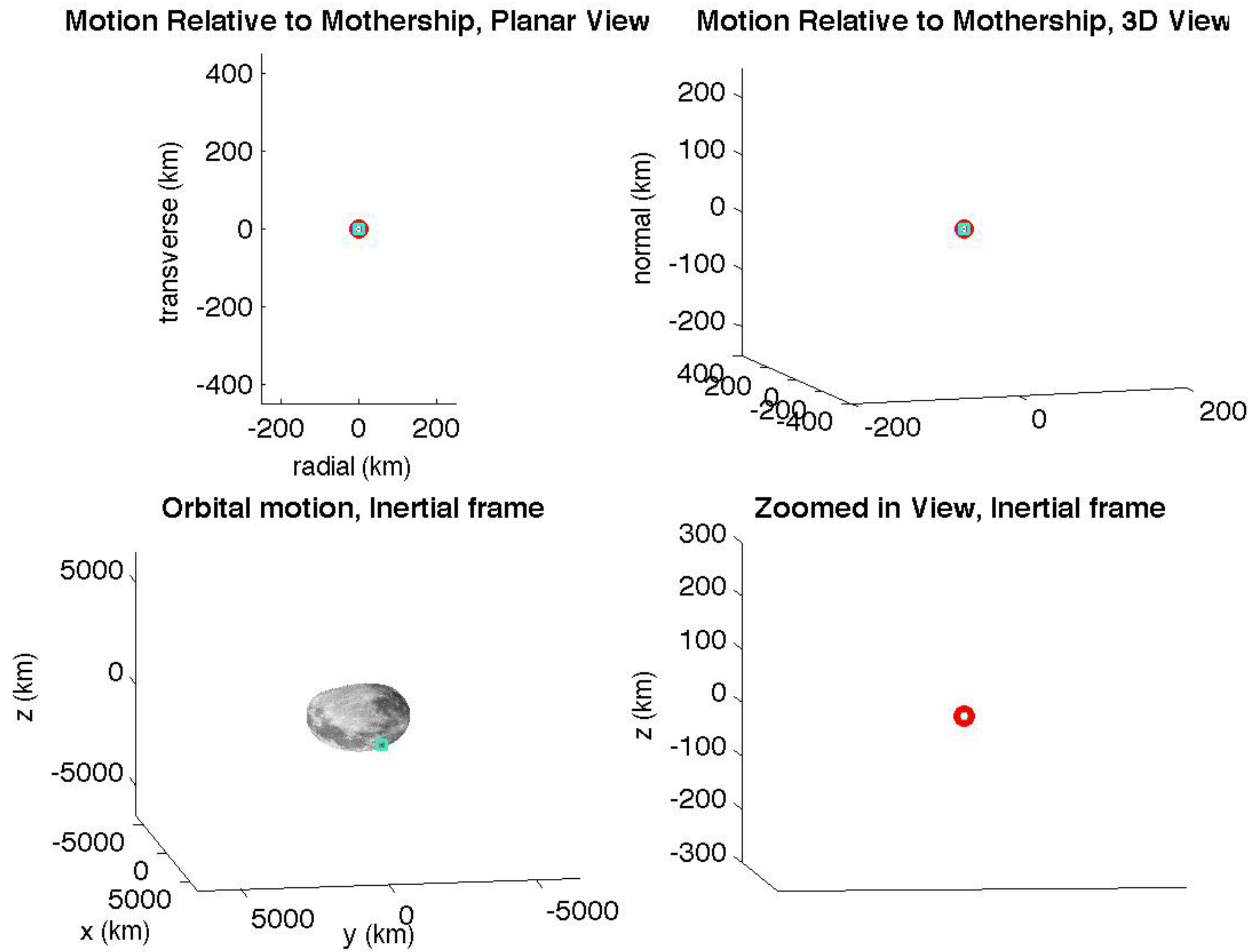
- ***Example:*** 4 spacecraft reconfigure to larger ring
  - Maneuver occurs at same location on relative orbit at ν=±90°
  - Reconfigure as many times within a ΔV budget





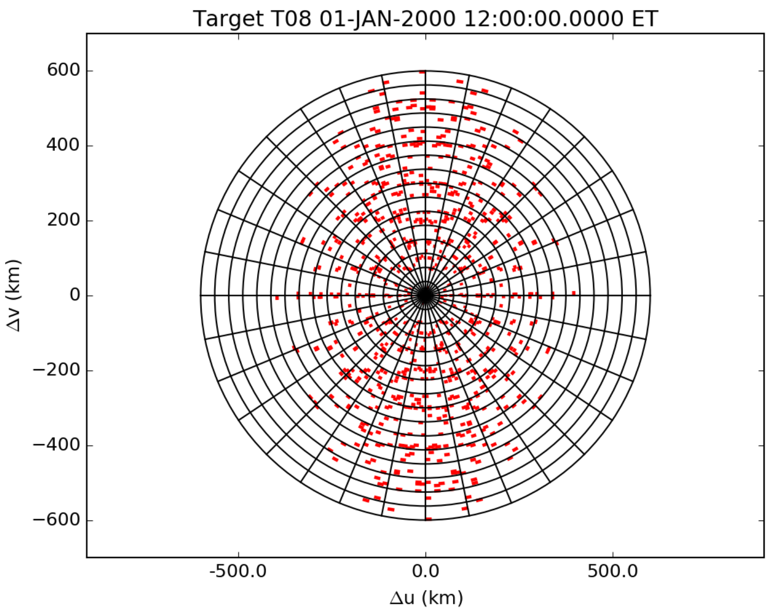


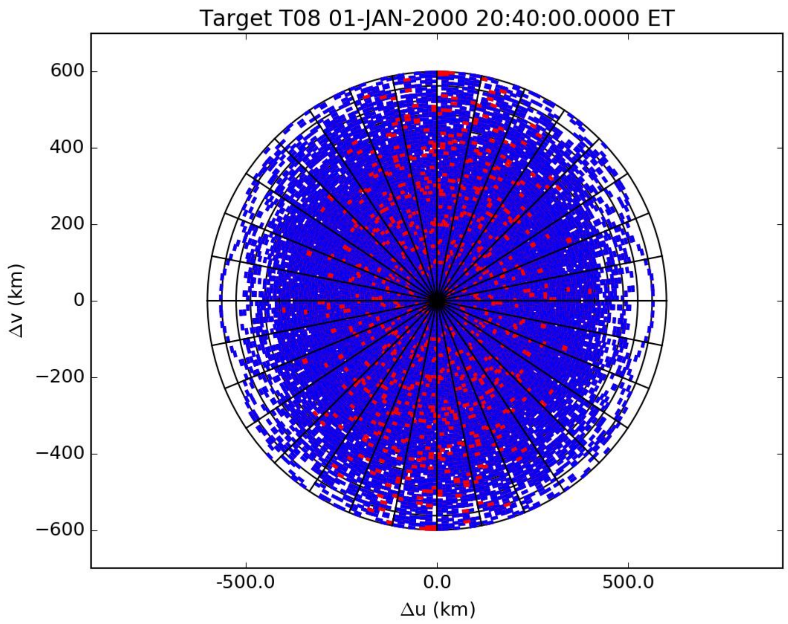
Full Mission Simulation



Baselines Achieved in the Celestial Sphere

- The formation design is driven by ***adequate coverage*** of collection targets
- Coverage is ***diversity of baselines*** formed by individual spacecraft pairs
- Baseline is projection of the relative position vector from one spacecraft to another, into the plane perpendicular to the direction of a target
- Example target at RA 0° and DEC 45° after final reconfiguration

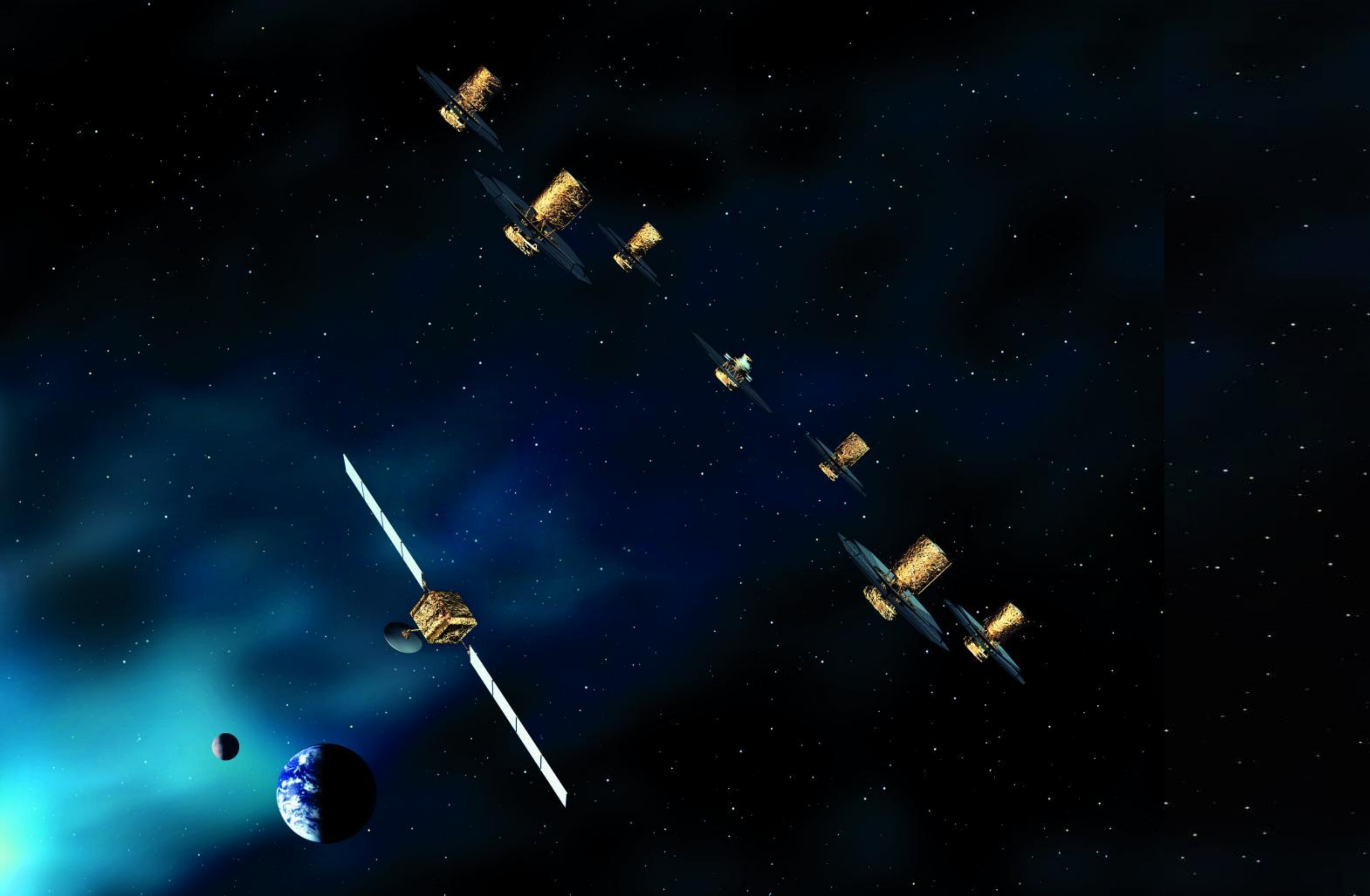




- For entire celestial sphere, constellation achieves 98% coverage.

Conclusion

- ***Networked constellations*** can enable novel types of missions, at lower costs and increased robustness
  - Spaced-based radio interferometers
- ***Constellation design methods***
  1. Constellation design using Linear Dynamics ***(CLD)***
     - Analytical solution, great for initial design stage
     - Can replicate linear design in nonlinear (two-body) model
     - Stationkeeping costs to account for higher-order dynamics
  2. Constellation design using Invariant Manifolds ***(CIM)***
     - Requires integration of state with STM
     - Higher-order dynamics can be included in design, avoiding large stationkeeping costs
- ***Operations***
  - Deployment
  - Reconfiguration to allow more science acquisition
    - Simple algorithm that can be implemented on board for autonomous ops.
    - Valid for low-thrust or high-thrust
  - Maximum baseline coverage



Jet Propulsion Laboratory